MATH-456 Discrete Dynamical Systems

Credit Hours: 3-0

Prerequisites: MATH-251 Ordinary Differential Equations

Course Objectives: The field of dynamical systems and especially the study of chaotic systems is one of the important breakthrough in science in this 20^{th} century. The purpose of the course is to introduce the ideas on discrete dynamical systems at the level of undergraduate students. The principal objectives of the course are to elaborate the elements of discrete dynamical systems and to consider particular systems with complex behavior.

Core Contents: Orbits, Graphical Analysis, Fixed and Periodic Points, Bifurcations, The Role ofthe Critical Orbit, Fractals

Detailed Course Contents: Orbits: Iteration, Orbits, Types of Orbits, The Doubling Function.Graphical Analysis: Orbit Analysis, The Phase Portrait.

Fixed and Periodic Points: Attraction and Repulsion, Calculus of Fixed Points, Periodic Points,Rates of Convergence.

Bifurcations: Dynamics of the Quadratic Map, The Saddle-Node Bifurcation, The Period-Doubling Bifurcation,The Transition to Chaos.

Chaos: Properties of a Chaotic System Manifestations of Chaos, Feigenbaum's Constant.

The Role of the Critical Orbit: The Schwarzian Derivative,The Critical Point and Basins ofAttraction, Newton's Method

Fractals: The Cantor Set, The Sierpinski Triangle, The Koch Snowflake, Topological Dimension,Fractal Dimension, Iterated Function Systems.

Course Outcomes: The students are expected to understand

- Mathematical aspects of theory of dynamical systems.
- The geometric aspects of discrete dynamical systems
- Bifurcations and chaos in discrete dynamical systems
- Fractals and fractal dimensions

Text Book: Robert L. Devaney, A First Course in Chaotic Dynamical Systems, Perseus Books Publishing, L.L.C., 1992.

Reference Books

1. Richard Holmgren, A First Course in Discrete Dynamical Systems, Springer, 1996
2. Mustafa R.S. Kulenovic, Orlando Merino, Discrete Dynamical Systems and DifferenceEquations with Mathematica, Chapman & Hall, 2002.
3. Rex Clark Robinson, An Introduction to Dynamical Systems: Continuous and Discrete,American Mathematical Socity, 2012.

Weekly Breakdown		
Week	***Section***	***Topics***
1	2.1, 2.4	Examples of Dynamical Systems from Finance, Ecology, Finding Roots and Solving Equations (Review).
2	3.1, 3.2	Iteration, Orbits.
3	3.3, 3.5	Types of Orbits, The Doubling Function.
4	4.1,4.2	Graphical Analysis, Orbit Analysis.
5	4.3, 5.1,5.2	The Phase Portrait, Fixed and Periodic Points, A Fixed Point Theorem, Attraction and Repulsion.
6	5.3-5.6	Calculus of Fixed Points, Periodic Points, Rate of convergence.
7	6.1-6.4	Bifurcations, Dynamics of the Quadratic Map, The Saddle-Node Bifurcation, The Period-Doubling Bifurcation.
8	7.1-7.3	The Quadratic Family, The Cantor Middle-Thirds Set.
9	**Mid Semester Exam**	
10	8.1,8.2	The Orbit Diagram, The Period-Doubling Route to Chaos.
11	10.1- 10.3	Three Properties of a Chaotic System, Other Chaotic Systems, Manifestations of Chaos.
12	12.1, 12.2	The Schwarzian Derivative, The Critical Point and Basins of Attraction.
13	13.1,13.2,	Newton's Method, Basic Properties, Convergence1 and Nonconvergence
14	14.1	Fractals, Chaos Game.
15	14.2,14.3	The Cantor Set Revisited. The Sierpinski Triangle.
16	14.4-14.6	The Koch Snowflake, Topological Dimension, Fractal Dimension.
17	14.6	Iterated Function Systems.
18	**End Semester Exam**	